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# THE NEW NUMBER OF THE COSMIC SAND.[1]

## ON THE EXTENT OF THE UNIVERSE AND THE LIMITS OF SPACE AND TIME.

ARCHIMEDES, one of the most original thinkers of antiquity, in a letter to Gelo, a son of King Hiero of Syracuse, discusses the curious question of the size of the universe. The title of his treatise reads: "On the Number of the Sand, or the Vastness of the Universe."

Calculations which are simply amazing when we take into consideration the general knowledge of the period brought Archimedes to a number for the aggregate of the smallest particles in the celestial globe (called sand) for whose expression it was necessary to invent new numerals. The following lines may perhaps serve to lift the hitherto impenetrable veil which has covered the motives of this document of Archimedes.

The sentence, *Quod non est in sensu non est in mundo* ("What does not exist in human consciousness [*Bewusstsein*] does not exist in the universe"), can be correctly understood only when we keep clearly in mind that there are three stages of human consciousness: There are sensations [*Empfindungen*]; there are concrete images [*Vorstellungen*[2]]; and there are concepts [*Begriffe*]. In these three instances the words "there are" [*es gibt*] have such different meanings that a large part of all differences of opinion in religion and philosophy can be referred to the different usage of the words, "there are (or exist)." All human beings possess the same mode of drawing conclusions logically. Differences of opinion arise either from lack of agreement in regard to the premises, or from the different use of exactly the same words, especially the words "there are."[3] The conflict of minds is at the same time a conflict of words.

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[1] Translated for *The Monist* from the German manuscript by Lydia G. Robinson.

[2] The word *Vorstellung* is very difficult to render properly into English. Perhaps it is most often translated by "idea" or "presentation"; but "idea" is too often used (as formerly by Locke) to cover practically any content of consciousness even including all three of the stages here mentioned, and "presentation" rather denotes an object as it is cognized at any given moment of

*There are sensations.* This reality not only does not require proof but is not even susceptible of proof, for none but our own sensations possess this certainty of reality. Sensations of other living beings are always inferred from motions by analogy with our own sensations. There are no sensations of other living beings in the sense that there are sensations of our own, but for these two very different realities our language possesses only the one expression, "there are."

The case is even worse with the content of our third stage of consciousness—ideal constructions or concepts [*Begriffe*]. Here we need only consider the controversies and proofs for and against the existence of God, and the dreadful tribulations which the want of understanding regarding the existence of concepts or ideal constructions has brought upon unnumbered people, in order to show the importance of the demonstrability of a content of consciousness. Again, there is freedom, or there is beauty, in quite a different sense from that in which we can say there are eyes or ears, and to endeavor to prove the existence of a concept, rather

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the perceptual process implying relation to a present object, which *Vorstellung* does not. Whereas the meaning of *Vorstellungen* may include three kinds of images, perceptual images, memory images, and imagined images, here—in its limitation to the second stage of consciousness—I follow Professors Tufts and Mead in defining it as "concrete images with their meanings."—Tr.

[3] Misunderstandings which, as frequently happens, arise from malice and the intention to hold up to contempt those who differ from us belong to the category of wasted energy.

than its applicability, is merely to display one's own want of understanding. The world of concepts lacks proof for the existence of its content as does the world of sensations, for proof belongs to the world of concrete images in their reference to human sense-impressions.

Our world, or reality, consists of those parts of consciousness which are at all times connected with human sensations, or can be regarded as so connected. We cannot prove directly that the stars actually exist because we cannot apply other sense-organs to corroborate the testimony of the eye. It is nevertheless enough for each person to believe that he would feel the heat of the stars if he came near enough to them, for him to believe that they really exist, instead of assuming that he is merely the victim of an optical illusion. We regard it as a further proof of the actual existence of the stars that we can predict by calculation what our eyes perceive and also the impressions recorded on photographic plates, observing that the predicted sensation and the sensation actually experienced agree closely, with but rare exceptions.

The world of concrete images, reality, is not only characterized by the fact that its parts can always be brought into direct connection with sense-impressions, but also by its constant capacity for relation with measure and number. Nothing in the world of reality is infinitely great or infinitely small; everything is circumscribed numerically. To count means to recall sensations. It is not the objects of the external world (as we call the computable part of the world of our consciousness) but our sensations that we count. Therefore it is only concrete images that can be counted and not concepts or ideal constructions, for they cannot be thought of as directly connected with sensations. The questions, how much soul, how much beauty, how much freedom there is, are inadmissible; on the other hand we can and must count concrete images.

Because concepts develop very gradually from concrete images in our consciousness as concrete images combine into larger unities, frequently without receiving any new appellations, the same word can easily denote either a concrete image or a concept.

A source (spring) in the mountains is a concrete image; a source meaning origin—for instance, the source of justice—is a concept. Again, a source in the historian's sense is a concrete image when he is designating a document as a source. The criterion is always whether or not we can connect the word in its intended acceptation with a sense-impression, or can think of it as so connected. If not we cannot count. The sources of a historical work can be counted, but not the sources of justice or religion. That our stages of consciousness have stages of transition which prove very troublesome in a discussion makes that discussion difficult even where there is the best intention on both sides to understand each other; just as the poverty of human language can only gradually meet the increasing demands made by the constant formation of new concepts by constructing new words.

If we ask about the extent of space and time, or, as many express it, the infinity of space and the eternity of time, we shall have to determine first of all whether space and time belong to concrete images or to concepts, or whether these words are after all used for both images and concepts. The last is the actual state of things. Space and time are directly connected with sense-impressions and depend on them. It is only by sense-impressions that we have a concrete image of space and time; therefore there can be no doubt that the words "space" and "time" belong to the second stage of consciousness, which is at all times connected with sense-impressions or can be regarded as so connected. By remembering the regular sequence of sense-impressions in motions we gradually construct our concrete images of our spatial world, the world of motive processes which give rise to sense-impressions. A single sense-organ cannot originate any spatial image; not until we have a combination of impressions of several senses is there produced in us the concrete image of a spatial world. Our so-called sense-organs, the eyes, ears, tongue, etc., contain many primitive sense-organs combined into a higher unity. Time, on the other hand, is a concrete image of a succession of sensations. A single sense-organ would suffice to provide us with a concrete image of time presupposing but one series of sensations and memory. Hence our concrete images of space and time involve a succession of sensations. The moment we drop human sensation space and time vanish as forms of our concrete images.

But space and time have not simply remained concrete images; they have developed farther into concepts which can no longer be thought of as connected with human sensation. We speak of four-dimensional and multi-dimensional spaces which have nothing in common with our concrete image of space but the word "space," without the slightest connection with human sense-impressions; we have within us a concept of time which is thought of as independent of the imageable course of human sensations. It is in this sense that man constructs also the unreal concept of eternity. Hence, there are concrete images of space and time that can and must be limited numerically, and there are the unlimited concepts of space and time which have nothing to do with numbers and therefore cannot count as actual existences in the world. We speak of actual existence only with regard to concrete images, leaving out of account the reality of sensations. We cannot ascribe actual existence to concepts, because by the word "actual" we mark the boundaries of the world of concrete images from all the surrounding world. Hence the question of the extent of the universe changes into that of the limitation of our concrete images of space and time.

In the world of concrete images—the world of concrete motor images, our world—there is nothing, as we have said before, that can be said to be infinitely small or infinitely great; everything is numerically circumscribed. How vast, then, is the extent of the starry universe which, without our will, leads us beyond the world of concrete images into the world of concepts and suggests to us the concepts of infinity and eternity? The question of the distance of the nearest stars was long since solved; the question of the distances of the farthest stars whose light will ever reach a human sense-organ is solved when we can estimate the transparency of the cosmic ether, which like everything else in the universe we know cannot be infinitely great.

Besides the hypothetical ether, interstellar space also contains multitudes of fine particles of matter—cosmic dust—which put a limit to our vision and construct a measurable celestial orb, just as atmospheric dust causes a blue dome to appear to surround us in the daytime. We must ask how far away a star, i. e., an illuminating body of approximately the dimensions of our own sun, must be for no perceptible ray of light to be able to reach our eyes. We possess a point of departure for the appreciation of this distance in the decrease in the number of visible stars beginning with a definite order of magnitude. If space were absolutely transparent to us the number of stars would increase the more (other things being equal) the less the intensity of their light appeared to us; generally speaking, the stars farthest away would give the weakest light. In fact, the number of stars does increase with the increased power of our optical instruments to stars of the eleventh magnitude, but then the number of stars giving a still feebler light decreases instead of increasing, as an indication of the obstruction of light in so-called empty space. Even were we to imagine all cosmic dust, all dark bodies, as absent from space, still we could not assume that the space between two stars is absolutely transparent—for we know there is nothing absolute in the physical world. By the amount of the obstruction of light in space the size of the space of our universe is determined; and this space is smaller than a sphere with a radius of $10^{30}$ centimeters according to the calculations of astronomers, who estimate a radius of $10^{21}$ cm. Therefore we are the better able to say that a sphere with a radius of $10^{100}$ cm is much greater than all the starry universe that will ever come into consideration for concrete human representation. An extension of our concrete images of space would only be conceivable if we could ascribe to the man of the future the ability to travel the distance of such a celestial orb, or more than $10^{70}$ cm from the earth. Leaving this out of the question, the imageable world-space is less than $10^{300}$ cubic centimeters.

The size of our universe to our imagination rests not only on the size of the space which a glimpse of the starry universe enables us to divine, but in spite of the poverty and limitation of our consciousness we can conjecture an unexhausted and inexhaustible abundance of multiplicity in cosmic events. In this respect also far narrower limits are placed to human imagination than we have hitherto suspected. With the finiteness of space the finiteness of time is indissolubly connected by the limitation of possible multiplicities. The more elements the world of concrete images in human consciousness is composed of the richer it is, but the human mind cannot do more than take these elements in turn. If the number of elements is not infinite then after a certain period the same sequence of cosmic events must recur. Nietzsche's theory of eternal rebirth finds its unshakable mathematical foundation in the finiteness of our actual space. The number of multiplicities which determine our world of concrete images is immeasurably smaller than the number of the smallest elements which can be thought of as connected with our senses when we think of our space as filled up with such elements. The question of the number and kind of the elements of our concrete images which actually exist will in the near future be seen to be one of the most important questions of the science of man.

The smallest elements that we can image as connected with a human sense-impression are those whose dimensions are not altogether too far removed from the dimensions of light-waves. On the basis of physical experiments we can assume that particles of $10^{-30}$ cm diameter cannot produce any sort of disturbances of light-waves. The smallest particles of the physicists have a diameter of about $10^{-9}$ cm. Small particles with a diameter of only $10^{-20}$ cm could not influence a human sense-impression to any extent. Their presence cannot be concretely imaged but only conceived. The number of such smallest particles in concretely imageable space, *the new number of cosmic sand,* would therefore amount to far less than $10^{300}/10^{-60} = 10^{360}$, while the old number of the sand computed by Archimedes would be $10^{64}$ when translated into our own numerals.

Although the new cosmic number is inconceivably greater in numerical value than that of Archimedes, yet in category it is not superior to his ancient number. To perceive a larger number of decimal places we do not require a new kind of multiplicity. Not until we calculate the duration of time from the new cosmic number are we led beyond the thought of Archimedes to new numerical categories for which we are obliged to invent new numerical terms.

Counting is an abbreviated recollection of a sequence of touch-impressions. The broader the range of our knowledge becomes, the greater will be the numbers with which we deal. The world war of 1914 with the enormous enrichment of the content of knowledge that it affords humanity has brought to civilized mankind an extraordinary increase of those numerical quantities which must be taken into consideration. Before the war the limit of the imagination of average humanity did not extend beyond millions. Now milliards[4] occupy the place of the former millions. When we realize that there are tribes whose numerals end with 20 (hands and feet) we shall not wonder that there arises from time to time a need for the expansion of numerical systems. A new larger numeral means greater facility in counting and in communicating large numbers to our fellow men. Since the value expressed by 10,000 (μύριοι) was, generally speaking, the highest numeral the Greeks were familiar with, Archimedes, in order to be able to express his "number of the sand" created new numerical periods by multiplying 10,000 by itself. The index of the last period which could be formed in this way would be $10^4$. Our present numerals form a series whose last easily pronounceable number with 600 places we would call a centillion. Higher numbers can be pronounced only with great loss of time. A million times a million is called a billion, and so on up to a centillion which is a million multiplied by itself one hundred times. A million is $10^6$, a centillion therefore $10^{600}$. We form our large numerals by multiplying by a thousand. A million is a thousand times a thousand. Our new cosmic sand number, which we have placed at $10^{360}$, though greater than the number of the smallest particles in our whole world of concrete images, is therefore much smaller than a centillion.

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[4] Though many scientific people in the United States have for some time been used to the term "billion" as denoting millions of millions, and "trillion" for a million of billions as is the common meaning of these terms in Europe, still the popular comprehension and general usage here has continued to employ "billions" for thousands of millions and "trillions" for thousands of these billions, etc. Mr. Friedenthal's exposition incidentally gives a forceful argument for entirely abandoning this popular traditional usage in our newspapers and elementary text-books in order that we may keep abreast with the line of inevitable progress of scientific knowledge. Beyond the hundred millions we can then speak of thousands of millions or join the English and Germans in adopting the French term *milliard*.—TR.

Instead of multiplying our numbers as Archimedes did we will raise them to higher powers, and to name the number raised to a power as great as itself we will use the suffix *-iode*; thus a million raised to the millionth power will be a *milliode,* and the largest number thus conveniently pronounceable (a centillion raised to the centillionth power) a *centilliode.* In figures we write a milliode $1{,}000{,}000^{1000000}$ or $10^{6(10^6)}$, a centilliode $10^{600(10^{600})}$. The number of variations of $n$ elements is the iode of $n$, or $n^n$. The iode of our new cosmic number, $10^{360(10^{360})}$, is greater than all the multiplicity of events in our actual world, but if we want to form a larger numeral than a centilliode we can call a number $n$ raised to the $n^n$th power an *iodade.* The iodades are also interesting from a technically numerical point of view, because they represent the highest series which can be formed with three identical numbers as the only elements. Hence we have the iode series $1^1$, $2^2$, $3^3$, $4^4$, to $n^n$, and the iodade series $1^{(1^1)}$, $2^{(2^2)}$, $3^{(3^3)}$, $4^{(4^4)}$, to $n^{(n^n)}$.

Of this second series of powers we can assert that no human imagination can make requirements which cannot be easily satisfied in the world of concrete images within this series. The largest number that is easily pronounced would be a centillion multiplied a centilliode times by itself, to be called a *centilliodade.*

The next higher series of powers would contain those numbers in which the base is raised to the power of its iodade, or $n[n^{(n^n)}]$. It would be entirely superfluous to give names to this series and those that follow. Mankind will scarcely be required to form new numerals to represent larger numbers than the new iode and iodade numerals.

In order to pass from the infinity of space to the infinity of time we shall take for our minimum unit of time the period required for a tiny particle with a diameter of $10^{-20}$ cm moving with a velocity of $10^6$ (a million) kilometers [$= 10^{11}$ cm] per second to pass in a straight line the length of its own thickness, i. e., in order to exchange places with the particle next to it. We have the equation $t = s/v$, in which the time is equal to the path divided by the velocity. This gives $10^{-20}/10^{+11}$, or $t = 10^{-31}$ seconds. Although this particle of time may be too small with respect to imageable particles of time, yet, on the other hand, the new cosmic number is so much greater than all imaginable possibilities that the period of time which is calculated from the variation of the cosmic number up to the exhaustion of all variations is computed as far too great with respect to the imageable duration of time. If we multiply the multiplicity of the universe by the minimum unit of time we arrive at the formula $(n^n)t = 10^{360\,(10^{360})}$ seconds. In this formula we can neglect $t$ without virtual inaccuracy. According to this formula it is practically of no consequence how long we regard the time up to the completed change of the world-aspect, i. e., from one variation to another. *The greatest duration of time imaginable is in every case very much smaller than the iode of the new cosmic sand number.* Yet before the expiration of this period a world-aspect must return which has been present before. Man will regard the same course of further processes as inevitably connected with this recurrence of the same situation, since we assume that the whole future is necessarily conditioned by the arrangement of the smallest particles in one definite moment of time.

It must be granted that owing to the distinctions of right and left in space and of forward and back in time one arrangement (or variation) of the smallest particles does not strictly determine the course of all future time. We may conceive of an arrangement in space reversed like a mirrored reflection but otherwise the same, and of a sequence of events in time that is retrogressive but otherwise the same. Both of these exceptions must be ignored in interpreting the above rule.

But even if we would regard all the variations of the world-aspect as entirely independent of each other, i. e., if we regard mere chance as the decisive factor in determining the series of variations, by the aid of the calculus of probabilities we must still inevitably infer a limited duration of imageable time from a limitation of imageable space. Of course the probability that in the accidental sequence of the variation of many elements not only does one situation repeat itself but the whole sequence is repeated, is slight but by no means infinitely small.

The probability that all $n^n$ variations of $n$ elements will be repeated in the same sequence, in spite of absolute independence of each other, is the reciprocal of the iodade of $n$, hence $1/n^{(n^n)}$. If we construct the iodade of our new cosmic number we shall obtain a numeral for a period of time that is greater than the probability of the whole sequence of cosmic events repeating itself in the same way, without any interdependence of variations. The expected time will therefore be much shorter than the period calculated as passing between one recurrence and the next, shorter than $10^{360}[10^{360}(10^{360})]$ seconds.

The following speculation may enable us to form a faint notion of the length of this period of time. There is a German fairy-tale in which the following answer is given to an inquiry as to the duration of eternity: Within a forest stands a mountain of purest diamond one mile long, one mile wide, and one mile high. Every thousand years a little bird comes, sings a song, and whets his bill against the diamond mountain. When the whole mountain has thus been whetted away so that nothing at all is left of it then the first second of eternity has passed. A simple calculation shows that there is no comparison between this second of eternity and the iodade of the new cosmic number, not even with the cosmic number itself, $10^{360}$. We must assume that at least one molecule of the diamond is lost in the process of whetting, otherwise nothing at all would be lost—which is impossible. A mountain a cubic mile in size does not contain as many as $10^{40}$ molecules (there are more than $10^{24}$ in a liter).[5] A thousand years contain a little more than $3 \times 10^9$ seconds. We need only multiply the number of seconds by the number of molecules to obtain a length of $10^{50}$ seconds, or about an octillion,[6] for this second of eternity. Even the cosmic number of Archimedes was therefore incomparably greater than the time which would actually pass in completing the whetting away of the diamond mountain. Of course it is clear that with friction a diamond would really wear away a much larger mass than the size of a molecule.

On the other hand, the iodade of the new cosmic number $10^{360}$ might very well claim to be regarded as the second of eternity since all the time the greatest human imagination can conceive is as nothing to this number, and no problem exists which the imagination of a naturalist of the present or future can propose, whose solution in numbers will not be very much smaller than the iodade of the new number of the cosmic sand.

*Sunt certi denique fines,* holds true of all events within our conceptions of space and time. Whoever is inclined to presumption, let him estimate the numbers of the different variable concrete elements within his own consciousness and compare their iodade with that of the new number of the cosmic sand.

HANS FRIEDENTHAL.

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[5] [In fact less than one twentieth that many, although of course German miles are thought of: $1\,m^3 = 10^3\,l$; $1\,km^3 = 10^9\,m^3 = 10^{12}\,l$; 1 cubic mile $= (7.5\,km)^3 = 421.875 \times 10^{12}\,l = 421.875 \times 10^{36}$, i. e., less than half $10^{39}$, molecules.—ED.]

[6] [To use the (approximate) values of note 5: $422 \times 10^{36} \times 3 \times 10^9 = 1266 \times 10^{45} = 1.3 \times 10^{48}$ seconds.—ED.]